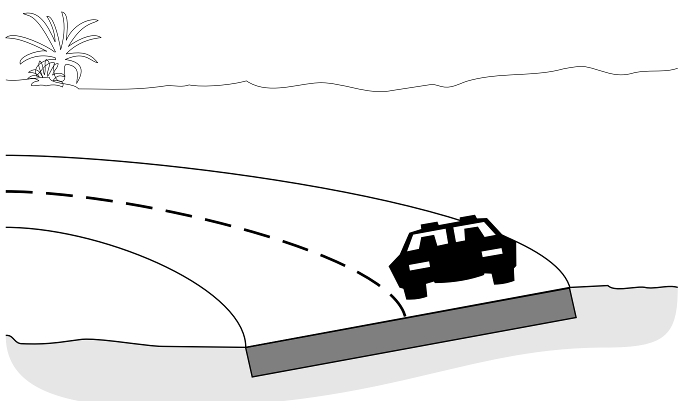
MISN-0-409

APPLICATIONS OF THE THEORY OF MOTION



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APPLICATIONS OF THE THEORY OF MOTION

by

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Title: **Applications of the Theory Of Motion**

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Input Skills:

1. Vocabulary: force (MISN-0-408).
2. State the direction and magnitude of a particle's gravitational acceleration due to the earth (MISN-0-406).
3. Calculate a vector's components parallel and perpendicular to a specified direction (MISN-0-407).
4. State the equation of motion for a particle (MISN-0-408).

Output Skills (Knowledge):

- K1. Vocabulary: weight, contact forces.
- K2. State four properties of the gravitational force.
- K3. Describe these contact forces in terms of their magnitude and direction: (a) force due to a spring; (b) tension force due to a string; (c) force due to a solid surface.
- K4. Describe tension forces due to a light string.

Output Skills (Problem Solving):

- S1. Identify and describe the forces acting on a given particle.
- S2. Apply the equation of motion to determine an object's mass, acceleration, or one of the forces acting on it, when: (a) individual forces are specified by arrow(s) on a grid or; (b) the acceleration and individual forces are all parallel or perpendicular to a given direction.
- S3. Relate qualitatively the motion of a particle along a straight path to the forces acting on it.
- S4. Use the reciprocal relation between mutual forces to apply the equation of motion to particles moving together with the same acceleration.

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MISN-0-409

APPLICATIONS OF THE THEORY OF MOTION

Abstract:
In this unit we shall apply the theory of motion to various simple situations frequently encountered in everyday life. We shall begin by examining some common forces. A knowledge of these forces will then permit us to use systematically the equation of motion $m\vec{a} = \vec{F}$ to understand and make predictions about many diverse situations.

SECT.

A LONG-RANGE AND CONTACT FORCE

To describe the forces encountered in everyday life, it is often not necessary to understand in detail how these forces result ultimately from the fundamental forces between individual atoms. Instead, it is usually quite sufficient to describe many common forces from an "empirical" point of view, i.e., to describe only their gross observable properties. (Such an empirical approach is useful in many fields. For example, considerable progress can be achieved in biology or medicine without the need to understand all the biochemical reactions responsible for biological functioning.)

From such an empirical point of view, it is useful to distinguish between two kinds of forces on one object due to another. The first kind of force is readily observable even if the separation between the interacting objects is appreciable. Such a force is called a "long-range" force. For example, a ball thrown upward near the surface of the earth is acted on by the downward "gravitational" force due to the earth (since the ball is accelerated in the downward direction), even if the ball is at a large distance above the surface of the earth. Similarly, a magnet experiences a "magnetic" force due to another magnet although the magnets do not touch each other. Hence such gravitational or magnetic forces are both long-range forces.

The other kind of force is observably large only if the separation between the interacting objects is of atomic size (i.e., about 10^{-10} meter). In other words, the force is appreciable only if the objects "touch" each other (i.e., if the separation between them is negligibly small from a large-scale point of view). Such forces, called "short-range" or "contact" forces, arise whenever any two objects touch each other. Examples are the force exerted on a nail by a hammer hitting the nail, the force exerted on a man by the chair on which he is sitting, or the force exerted on a chandelier by the chain from which it is suspended.

Contact forces are due to the forces which nearby atoms exert on each other. For example, consider the force $\vec{F}$ on some object A by some other object B with which it is in contact (e.g., on a person A by a mattress B on which he is lying, as shown in Fig. A-1). Then the force $\vec{F}$ on A is due to the interaction between the atoms near the surface of A with the adjacent atoms near the surface of B, i.e., the force $\vec{F}$ is really the

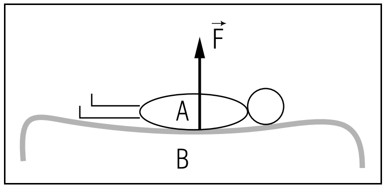


Fig. A-1: Contact force $\vec{F}$ exerted on a person A by a mattress B on which he is lying.

force exerted on the *surface* of A by the *surface* of B. Note, however, that atoms near the surface of either object interact also with adjacent atoms slightly further from the surface, that these atoms interact in turn with adjacent atoms still further from the surface, and so on. Hence the contact force $\vec{F}$ on one object due to the other involves *indirectly* the interaction between *all* adjacent atoms inside the objects and depends therefore on the separations between adjacent atoms throughout the objects. Thus the contact force $\vec{F}$ increases in magnitude as the objects are "deformed" (i.e., as the separations between the adjacent atoms in them are changed from their normal values) and has a direction opposing their further deformation. (For example, in Fig. A-1 the force $\vec{F}$ exerted on the person by the surface of the mattress increases as the mattress is compressed and has an upward direction opposing this compression.)

To identify *all* the forces acting on some object, we must identify all the other objects with which it interacts both by long-range forces and by contact forces. The long-range forces comprise often only the gravitational force due to the earth. The contact forces are due to all the other objects with which the object is in contact. In the next two sections we shall discuss the properties of these forces.

Identifying Individual Forces (Cap. 1)

A-1 A man's hand holds a large rubber eraser and rubs it across the surface of a notebook. What objects exert forces on the eraser? *(Answer: 104)*

A-2 A child swings from a rope tied to a tree branch. Which of the following objects exert a force on the child: the rope, the tree branch, the earth, the child? *(Answer: 107) (Suggestion: [s-6])*

SECT.

B GRAVITATIONAL FORCE NEAR THE EARTH

As discussed in text section E of Unit 406, any particle falling near the earth under the sole influence of gravity has a downward acceleration $\vec{g}$ which is independent of all properties of the particle. The value of this "gravitational acceleration" $\vec{g}$ is nearly constant in any region of linear size much smaller than the radius of the earth (although it decreases slowly with increasing distance from the earth).

According to the definition of force, a particle of mass m moving with an acceleration $\vec{a}$ is acted on by a force $\vec{F} = m\vec{a}$. Since $\vec{a} = \vec{g}$ for any particle subject only to the gravitational interaction with the earth, any such particle must then be acted on by a "gravitational force" $\vec{F}_g$ due to the earth such that: *

$$\boxed{\vec{F}_g = m\vec{g}} \tag{B-1}$$

* Here and in the following sections we shall use a single subscript to indicate the interaction or object responsible for the force.

The properties of this gravitational force $\vec{F}_g$ are then apparent from those of the gravitational acceleration $\vec{g}$.

(a) The gravitational force is a long-range force (since it acts on the particle even when this particle is at a considerable distance from the surface of the earth).

(b) The gravitational force is attractive (since it is downward, i.e., toward the center of the earth).

(c) The gravitational force depends on the mass of the particle but on none of its other properties (since g is independent of all properties of the particle).

(d) The gravitational force is nearly constant within any region of linear size much smaller than the radius of the earth, but decreases slowly with increasing distance from the earth (since $\vec{g}$ has the corresponding properties).

The word “weight” is commonly used in accordance with this definition:

Def. | **Weight**: The weight of an object is the *magnitude* of the gravitational force on this object. | (B-2)

Thus Eq. (B-1) implies that the weight w of an object of mass m is simply $w = |\vec{F}_g| = mg$. For example, near the surface of the earth where $g = 10\,\text{meter/sec}^2$, the weight w of a newborn baby having a mass $m = 4\,\text{kg}$ is $mg = (4\,\text{kg})(10\,\text{meter/sec}^2) = 40\,\text{newton}$. Note that the weight of an object, unlike its mass, depends not only on the properties of the object, but also on the location of the object relative to the earth which exerts the gravitational force. Thus the weight of an object depends on g and decreases slightly with increasing height of the object above the surface of the earth.

The gravitational force $\vec{F}_g$ on a particle due to the earth depends, like all other forces, only on the relative positions of the interacting objects. Hence the result $\vec{F}_g = m\vec{g}$ ought to be *always* true, irrespective of how the particle moves (i.e., irrespective of whether the particle falls downward, or moves along a trajectory like a projectile, or is lying on the floor).

The equation of motion $m\vec{a} = \vec{F}$ for any particle of mass m moving under the *sole* influence of gravity due to the earth can thus always be written as

$$m\vec{a} = m\vec{g}$$

Dividing both sides by m gives then

$$\vec{a} = \vec{g} \tag{B-3}$$

Thus we arrive at the result (already used in units 406 and 407) that the acceleration of any particle moving under the sole influence of gravity is always equal to $\vec{g}$, irrespective of how the particle moves.

Describing the Gravitational Force (Cap. 1)

$\boxed{\text{B-1}}$ What is the gravitational force due to the earth on each of the following objects: (a) A kitten of mass 1 kg during a jump near the earth’s surface. (b) The same kitten at rest on the ground. (c) A satellite of mass $5 \times 10^4\,\text{kg}$ in a circular orbit just above the earth’s surface. *(Answer: 101)*

Knowing About Weight and Mass

$\boxed{\text{B-2}}$ Do each of the following phrases describe the quantity weight or the quantity mass? (a) Can be written using the unit kilogram. (b) Can be written using the unit newton. (c) Is the magnitude of a force. (d) Is always the same for a given object. (e) For a given object, is different when the object is at sea level than when it is on a high mountain. *(Answer: 110)*

$\boxed{\text{B-3}}$ A bar of gold is bought in Cape Town, South Africa (where the gravitational acceleration has the magnitude $g = 9.797\,\text{m/s}^2$) and sold in Stockholm (where g is $9.818\,\text{m/s}^2$). In which city does the gold have a larger weight? In which city has it a larger mass? *(Answer: 112)*

SECT.

C PROPERTIES OF SOME CONTACT FORCES

In Sec. A we noted that the contact force produced when an object touches another object depends on the deformation of the objects and tends to oppose this deformation. Let us now examine several examples of such contact forces.

FORCE DUE TO A SPRING

Consider a particle (e.g., a golf ball) attached to the end of a spring as shown in Fig. C-1. Then a contact force $\vec{F}_s$ acts on the particle due to the end of the spring. This force depends on the deformation of the spring, i.e., on the change of length $(L - L_0)$ of the spring, where L is the actual length of the spring and L_0 is its normal undeformed length.

The direction of $\vec{F}_s$ is such as to oppose the deformation of the spring. Thus the force $\vec{F}_s = 0$ if the spring has its normal undeformed length L_0 (as shown in Fig. C-1a). If the spring is stretched, the force $\vec{F}_s$ tends to restore the particle to the position where the spring is undeformed, i.e., $\vec{F}_s$ has a direction *toward* the spring as shown in Fig. C-1b. If the spring is compressed, the force $\vec{F}_s$ again tends to restore the particle to the position where the spring is undeformed, i.e., $\vec{F}_s$ has a direction *away* from the spring as shown in Fig. C-1c.

The magnitude of the force $\vec{F}_s$ increases as the deformation of the spring increases, i.e., as the magnitude $|L - L_0|$ of its change of length increases.

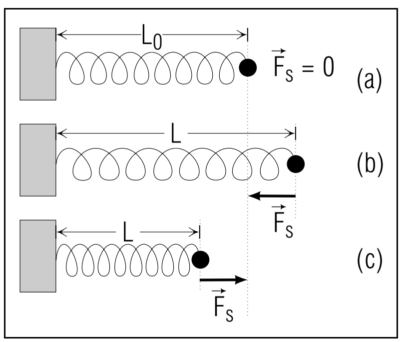


Fig. C-1: Force $\vec{F}_s$ exerted on a particle by a spring for various deformations of the spring. (a) Normal undeformed length L_0 of the spring. (b) Stretched spring. (c) Compressed spring.

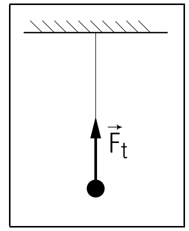


Fig. C-2: Tension force $\vec{F}_t$ exerted on a particle by a string.

Thus the magnitude of $\vec{F}_s$ is zero when $L = L_0$ and increases either when the spring is stretched so that L becomes larger than L_0, or when the spring is compressed so that L becomes smaller than L_0.

The actual dependence of the magnitude of $\vec{F}_s$ on the deformation of the spring depends on the particular spring under consideration. If the deformation is sufficiently small, the magnitude of $\vec{F}_s$ is proportional to $|L - L_0|$. Then $|\vec{F}_s| = k|L - L_0|$, where k is a constant (the "spring constant") which is characteristic of the particular spring.

FORCE DUE TO A STRING

Consider a particle attached to the end of a string. The contact force exerted on the particle by the end of the string has then a direction along the string and *away* from the particle (as indicated in Fig. C-2). A force having such a direction is called a "tension" force and can be denoted by $\vec{F}_t$. *

> * A string is thus unlike a rod which has "stiffness." For example, a rod can exert a sideways force perpendicular to the rod, or a force toward the particle as well as away from the particle (i.e., a "pushing" force as well as a "pulling" force).

The tension force $\vec{F}_t$ is due to the deformation of the string and is larger for larger deformations of the string. However, a large tension force may often be produced by a deformation which is negligibly small (i.e., which is only observable by sensitive methods).

FORCE DUE TO A SOLID SURFACE

Consider a particle A which is in contact with the surface of some solid object B (e.g., a package A sliding across the surface of a table B).

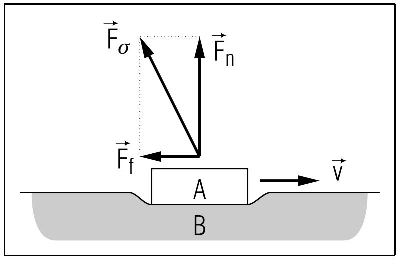


Fig. C-3: Force $\vec{F}_\sigma$ exerted on a particle A by the surface of an object B with which the particle is in contact.

The contact force exerted on the particle by the solid surface can be denoted by $\vec{F}_\sigma$. *

* The subscript σ (Greek letter "sigma") indicates the solid surface.

As usual, this contact force is due to the interaction between the atoms of A and B near the surface and is accompanied by a deformation of the object (as indicated in Fig. C-3 or by the mattress in Fig. A-1). However, the magnitude of the contact force may be quite large even if the deformation is negligibly small.

The contact force $\vec{F}_\sigma$ may have various directions relative to the surface. Hence it is convenient to express it in terms of its component vectors perpendicular and parallel to the surface by writing

$$\vec{F}_\sigma = \vec{F}_n + \vec{F}_f \qquad \text{(C-1)}$$

Here the component force $\vec{F}_n$ perpendicular to the surface is commonly called the "normal" force (since the word "normal" is often used to mean the same as "perpendicular"). The component force $\vec{F}_f$ parallel to the surface is commonly called the "frictional" force. The properties of these two component forces can be inferred from some simple observations.

The normal force $\vec{F}_n$ perpendicular to the surface opposes the deformation of the surface and has therefore a direction *away* from the surface (if we neglect unusual situations where the surface is sticky or covered with glue).

The frictional force $\vec{F}_f$ can have *any* direction *parallel* to the surface, although its direction is always such that $\vec{F}_f$ resists the motion relative to the surface of the points of the particle in contact with the surface. The magnitude of the frictional force depends on the roughness of the surface.

If the surface is smooth (like the smooth surface of a sheet of ice), the frictional force may be small enough to be neglected. In this case the force

$\vec{F}_\sigma$ exerted on the particle by the surface has a negligibly small component vector $\vec{F}_f$ parallel to the surface, i.e., it is simply perpendicular to the surface.

The magnitude F_f of the frictional force is found to increase with increasing magnitude F_n of the normal force. Usually, the magnitudes of these forces are merely proportional to each other, i.e., one can write

$$F_f = \mu F_n \qquad \text{(C-2)}$$

where the constant number μ is a quantity characteristic of the particular surfaces in contact with each other and is called the "coefficient of friction." (The value of this coefficient is usually somewhat larger when the particle is at rest relative to the surface than when it is moving relative to it.)

SIMULTANEOUS ACTION OF SEVERAL FORCES

Most objects near the surface of the earth are acted on simultaneously by the gravitational force due to the earth and by several contact forces. These forces may then be compared in useful ways, as illustrated by this example:

Example C-1: Measuring weight with a scale

A simple scale, such as a bathroom scale, consists essentially of a platform supported by a spring. As the spring is compressed, its change of length is indicated by a pointer moving around a graduated dial and provides thus an indication of the force $\vec{F}_s$ exerted on an object supported by the scale. Suppose that a woman of mass m stands on such a scale. How then is the force $\vec{F}_s$ indicated by the scale related to the weight of the woman?

The woman standing on the scale is illustrated in Fig. C-4. She is acted on by the downward gravitational force $\vec{F}_g = m\vec{g}$ due to the earth and by the force $\vec{F}_s$ due to the scale.

The total force on the woman is then the vector sum of these two forces. If the acceleration of the woman is $\vec{a}$, the equation of motion of the woman is then

$$m\vec{a} = \vec{F}_g + \vec{F}_s \qquad \text{(C-3)}$$

Suppose that the woman and the scale are at rest relative to the earth (which is approximately an inertial frame). Then the woman's

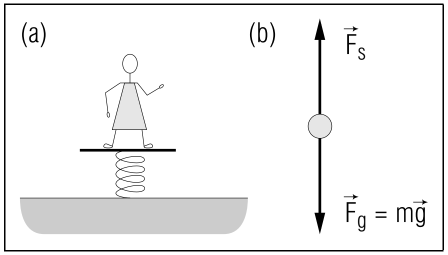


Fig. C-4: Measurement of weight by a scale. (a) Woman standing on the scale. (b) Forces acting on the woman.

acceleration $\vec{a} = 0$ and Eq. (C-3) implies simply that

$$0 = \vec{F}_g + \vec{F}_s$$

so that

$$\vec{F}_s = -\vec{F}_g = -m\vec{g} \tag{C-4}$$

On the other hand, suppose that the woman is standing on the scale in an elevator moving with a vertical acceleration $\vec{a}$. Then the acceleration of the woman is also $\vec{a}$. Hence Eq. (C-3) implies that

$$\vec{F}_s = m\vec{a} - \vec{F}_g = m\vec{a} - m\vec{g}$$

or

$$\vec{F}_s = m(\vec{a} - \vec{g}) \tag{C-5}$$

In the first case where the woman is at rest relative to the earth so that $\vec{a} = 0$, Eq. (C-4) shows that the *magnitude* $|\vec{F}_s|$ of the force exerted by the scale is simply equal to the weight $w = mg$ of the woman. On the other hand, Eq. (C-5) shows that magnitude $|\vec{F}_s|$ of the force indicated by the scale is certainly *not* equal to the weight mg of the woman when she is accelerated.

The weight w of an object (i.e., the magnitude of the gravitational force $\vec{F}_g$ on it) is simply proportional to its mass since $w = |\vec{F}_g| = mg$. Hence the ratio w/w' of the weights of two objects A and A' at the same place is merely equal to the ratio m/m' of their masses. Thus one can compare the masses of two objects by merely comparing their *weights* (e.g., by using a scale on which the objects rest in mechanical equilibrium). For objects of everyday size, this method of "weighing" (i.e., of comparing gravitational forces) provides a far more convenient and precise way of

comparing masses than the comparison of accelerations described in text section C of Unit 408.

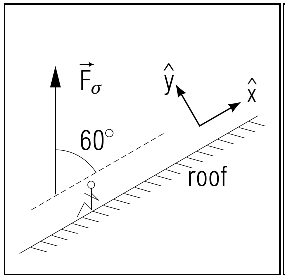


Fig. C-5.

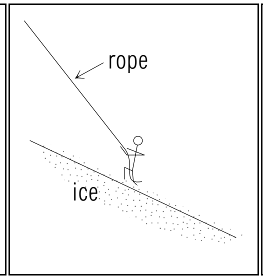


Fig. C-6.

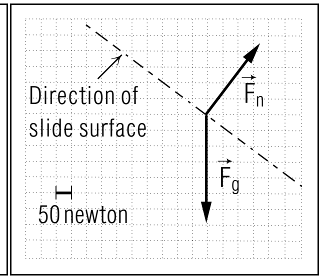


Fig. C-7.

Knowing About Contact Forces

C-1 (a) In Fig. C-1b, suppose that the particle is pulled to the right, extending the spring farther. Does the magnitude of $\vec{F}_s$ become larger or smaller? What is the direction of $\vec{F}_s$? (b) In Fig. C-1c suppose that the particle is pulled very slightly to the right (so that L remains less than L_0). Does the magnitude of $\vec{F}_s$ become larger or smaller? What is the direction of $\vec{F}_s$? *(Answer: 108)*

C-2 A carpenter sitting on a roof is acted on by a force $\vec{F}_\sigma$ due to the roof surface (Fig. C-5). If $\vec{F}_\sigma$ has a magnitude of 800 newton, what is the normal force $\vec{F}_n$ and the frictional force $\vec{F}_f$ on the carpenter due to the surface of the roof? *(Answer: 102) (Suggestion: [s-14]) (Practice: [p-1])*

Describing Individual Forces (Cap. 1)

C-3 A child slides downward along the surface of a playground slide. (a) List the objects exerting forces on the child. (b) What algebraic symbols are commonly used to represent these forces? (Choose symbols for the *two* component forces due to the surface.) (c) Using a sketch showing the slide surface, draw and label arrows indicating the direction of each force. *(Answer: 105) (Suggestion: [s-10])*

C-4 A climber is held at rest by a rope while he is on an icy surface sufficiently smooth that the frictional force is negligible. (Fig. C-6) Choose common algebraic symbols for each force acting on the climber. Then using a sketch showing the icy surface and the rope, draw and label arrows indicating the direction of each force. *(Answer: 106)*

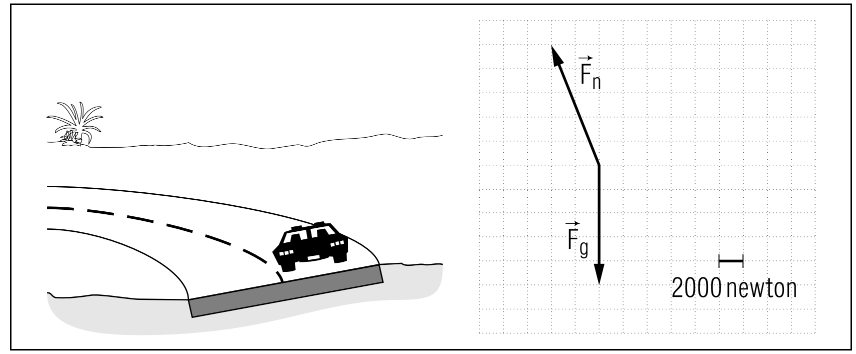


Fig. C-8.

Applying the Equation of Motion (Cap. 2a)

C-5 If the playground slide described in problem C-3 is very smooth, the forces on a sliding child of mass 35 kg are approximately those shown in Fig. C-7. What is this child's acceleration? *(Answer: 109) (Suggestion: [s-8])*

C-6 Suppose the car, shown from the rear in Fig. C-8, travels away from the viewer with a constant speed along a slippery, banked road which curves towards the left. The forces on the car are then the normal force $\vec{F}_n$ and the gravitational force $\vec{F}_g$ shown in Fig. C-8. If the car has a mass of 1.0×10^3 kg, what is its acceleration? If the car moves along a circular path of radius 100 meter, what is its speed? *(Answer: 103) (Suggestion: [s-2]) More practice for this Capability: [p-2], [p-3]*

SECT.

D APPLYING THE EQUATION OF MOTION

According to our theory of motion, every problem involving a system of particles in motion (or at rest) can be solved by applying the equation of motion $m\vec{a} = \vec{F}$ to each particle. Hence this equation provides the basis for an enormous number of practical applications.

The equation of motion $m\vec{a} = \vec{F}$ can also be applied directly to any large object if it can be considered as a single particle, i.e., if its motion can be adequately specified by a single velocity (because all its constituent particles move with the *same* velocity). For example, the equation of motion could be applied to an entire crate *sliding* along the floor, but not to a rotating wheel (because all parts of such a wheel do not move with the same velocity).

A STRATEGY FOR PROBLEM SOLVING

Whenever one is trying to apply general principles (such as the equation of motion) to solve specific problems, it is easy to make mistakes or to get lost in a maze of details unless one proceeds in a systematic way. Hence it is useful to follow a strategy which can guide one through the successive steps required to achieve a solution. A helpful general strategy applicable to any problem consists of the following major steps: (1) Describing the problem in useful terms; (2) Planning by selecting applicable principles and specifying how they are to be used; (3) Implementing the actual solution of the problem; and (4) Checking that the problem has been solved correctly.

The following statements describe this general strategy in greater detail and indicate specifically (in square brackets) how the strategy applies to problems involving the motion of particles.

1. Description
   a. Draw a diagram illustrating the situation.
   b. Summarize the known and desired information in terms of convenient symbols (including unit vectors to specify directions).

      [Express $m\vec{a} = \vec{F}$ in terms of symbols for the individual forces.]
2. Planning

   a. Decide on the principles to be used [$m\vec{a} = \vec{F}$].
   b. Choose the system to be considered [some specific particle]
   c. Express the principles in terms of known and desired information.

      [Express each of the quantities in the resulting equation in terms of symbols for known and desired information, using a diagram to indicate directions.]
3. Implementation
   a. Solve the equations to express the desired quantities in terms of symbols for the known quantities. (It may be useful to decompose vectors into their components along convenient directions.)
   b. Find desired numerical values by replacing symbols by their known values (including units, signs, and directions.
4. Checking
   a. Check that each step is correct.
   b. Check that the results are sensible (e.g., that the units are correct, that the signs and directions make sense, that the magnitudes are reasonable, . . .)

After the solution of the problem has been obtained, it is useful to do two more things: (1) To examine whether the solution might not have been obtained in different or simpler ways; and (2) to explore the implications of the results obtained so as to extend one's knowledge for future purposes.

Let us now illustrate the preceding general strategy in the case of a typical problem involving the motion of particles.

Example D-1: Car towed by a rope

A disabled car having a mass of 1.0×10^3 kilogram is towed along a horizontal straight road by a horizontal rope attached to the rear of a truck. The frictional force on the disabled car has a magnitude of 2×10^2 newton and the maximum tension force which can be exerted by the rope without breaking is 1.5×10^3 newton (i.e., about 300 pound). What then is the corresponding maximum acceleration with which the car can be towed along the road?

Description: Fig. D-1a illustrates the situation. We know: The mass of the car is $m = 1.0 \times 10^3$ kg. The tension force on the car exerted by

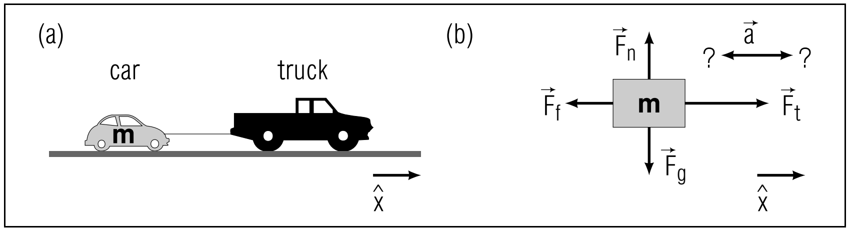


Fig. D-1: Car towed along a road. (a) Sketch of the situation. (b) Diagram indicating the forces on the car and its acceleration.

the rope is $\vec{F}_t = (1.5 \times 10^3\,\text{newton})\hat{x}$, if $\hat{x}$ denotes the horizontal direction from the towed car to the pulling automobile. The frictional force if $\vec{F}_f = -(2 \times 10^2\,\text{newton})\hat{x}$, opposite to $\hat{x}$ since it opposes slipping of the wheels along the road. The acceleration $\vec{a}$ of the car has a direction parallel to the straight road along which the car moves. We should like to find the value of this acceleration.

Planning: We want to apply the equation of motion $m\vec{a} = \vec{F}$ to the towed car since we are interested in its acceleration. To find the *total* force $\vec{F}$ on the car, we need to identify the individual forces on this car. These forces are the downward gravitational force $\vec{F}_g = m\vec{g}$ due to the earth and the following contact forces due to objects touching the car: the tension force $\vec{F}_t$ due to the rope and the force due to the surface of the road. This last force consists of the normal force $\vec{F}_n$ perpendicular to the road and of the frictional force $\vec{F}_f$. Thus the equation of motion $m\vec{a} = \vec{F}$ becomes

$$m\vec{a} = \vec{F}_g + \vec{F}_t + \vec{F}_n + \vec{F}_f\,. \tag{D-1}$$

Figure D-1b indicates the directions of all these forces and the possible directions of the acceleration $\vec{a}$.

Implementation: To solve the vector Eq. (D-1), we note that the equality of the vectors on both sides of this equation implies the equality of the corresponding component vectors parallel to any direction. The equality of the horizontal component vectors in Eq. (D-1) then implies that the horizontal component vector of the acceleration (i.e., the horizontal acceleration a itself) is related to the sum of the horizontal forces so that

$$m\vec{a} = \vec{F}_t + \vec{F}_f \tag{D-2}$$

Similarly, the equality of the vertical component vectors in Eq. (D-1) implies that the vertical component vector of the acceleration (which is 0 since $\vec{a}$ is horizontal) is related to the sum of the vertical forces so that

$$0 = \vec{F}_g + \vec{F}_n \tag{D-3}$$

We can now solve for the desired acceleration by using Eq. (D-2). Thus

$$\vec{a} = \frac{\vec{F}_t + \vec{F}_f}{m}$$

For the maximum permissible tension force of 1.5×10^3 newton, we then find the following corresponding numerical value of the acceleration

$$\vec{a} = \frac{(1.5 \times 10^3\,\text{newton})\hat{x} + (-2 \times 10^2\,\text{newton})\hat{x}}{1.0 \times 10^3\,\text{kg}}$$

or

$$\vec{a} = \frac{(1.3 \times 10^3\,\text{newton})\hat{x}}{1.0 \times 10^3\,\text{kg}} = (1.3\,\text{meter/sec}^2)\hat{x}$$

Checking: The units are correctly those of an acceleration. As expected, the direction of the acceleration $\vec{a}$ of the car is along the force $\vec{F}_t$ pulling the car and the magnitude of $\vec{a}$ seems reasonable. Furthermore, the Eq. (D-3) also makes sense since it implies that $\vec{F}_n = -\vec{F}_g$ so that the upward normal force on the car due to the road merely balances the downward gravitational force due to the earth.

Now: Go to tutorial section D.

Applying the Equation of Motion (Cap. 2b)

Demonstrating this capability includes using the equation of motion to produce systematic problem solutions which are clear to other people.

The term “known quantities” includes: numbers, all quantities specified in a problem, and symbols with known values, such as g and π.

$\boxed{\text{D-1}}$ The driver of a car traveling east applies his brakes. The horizontal road surface then exerts on the car a frictional force $\vec{F}_f = 3.0 \times 10^3$ newton west. If the car has a mass $m = 1.5 \times 10^3$ kg, what is the acceleration $\vec{a}$ of the car, and what is the normal force $\vec{F}_n$ exerted on the car by the road surface? (First express these quantities in terms of symbols for known quantities. Then find their values.) *(Answer: 111) (Suggestion: [s-5])*

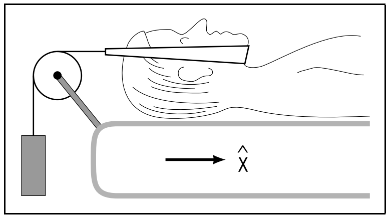


Fig. D-2.

D-2 The horizontal surface of a truck bed supports a crate of mass 1.5×10^2 kg. As the truck drives off, the truck and crate move with an acceleration $\vec{a} = 1.2\,\mathrm{m/s^2}\hat{x}$, where $\hat{x}$ is a horizontal unit vector directed along the road. What are the frictional and normal forces which the surface of the truck bed exerts on the crate? (Express your answers by using $\hat{x}$ and the upward unit vector $\hat{y}$). *(Answer: 115)*

D-3 The mass of a lamp suspended by a cord in an elevator cannot exceed a maximum safe value without danger of breaking the cord as the elevator accelerates. To determine this maximum safe mass m, suppose that while slowing down at the end of a descending trip, the elevator and lamp have their maximum upward acceleration of magnitude $\vec{a} = 1.0\,\mathrm{m/s^2}$, and that the cord exerts an upward force of magnitude $\vec{F}_t = 22$ newton, the largest force which the cord can exert without breaking. (a) Write the equation of motion for the lamp using the symbols provided. (b) Express m in terms of symbols for known quantities. (c) What is the value of m? *(Answer: 119) (Suggestion: [s-15])*

D-4 When the elevator and lamp described in problem D-3 move upward with constant speed, what is the value of the tension force exerted on the lamp by the cord? *(Answer: 122) (Suggestion: [s-1])*

D-5 A man with a neck injury lies with his 4.0 kg head supported at rest on the horizontal surface of a bed. The head is also acted on by a horizontal tension force, of magnitude 60 newton, due to a chin strap attached to a cord and weight (Fig. D-2). (a) If the bed surface exerts a frictional force $\vec{F}_f = 7.0$ newton $\hat{x}$ on the head, what is the force $\vec{F}_v$ on the head due to the cervical vertebrae of the neck? (b) *Review:* What is the force exerted by the *head* on the injured neck? (c) If the patient lifts his head slightly so that $\vec{F}_f$ is zero, what then is the value of $\vec{F}_v$? (To avoid such a change in $\vec{F}_v$, a patient's head is often placed on a small wheeled platform so that $\vec{F}_f$ is always negligible.) *(Answer: 117) More practice for this Capability: [p-4], [p-5]*

SECT.

E COMPONENT EQUATIONS OF MOTION

As illustrated by the example in the preceding section, the equation of motion $m\vec{a} = \vec{F}$ implies a corresponding equation relating the component vectors of the acceleration $\vec{a}$ and of the total force $\vec{F}$ parallel to *any* direction. Hence problems of motion can often be greatly simplified by dealing with component vectors parallel to some convenient directions. For example, if a particle moves along some specified path, its motion can usually be discussed most readily by considering the component vectors of the forces parallel to this path.

Example E-1: Sled sliding down a hill

A sled is sliding down a snow-covered hill which makes an angle with the vertical direction of the gravitational force. What is the acceleration of the sled along the hill if the frictional force on the sled is negligibly small?

► *Description*

Fig. E-1a illustrates the situation where we know the angle θ between the downhill direction $\hat{x}$ and the vertically downward direction. We want to find the acceleration $\vec{a}$ of the sled along the hill.

► *Planning*

We can apply the equation of motion $m\vec{a} = \vec{F}$ to the sled. The total force $\vec{F}$ on the sled consists of the gravitational force $\vec{F}_g$ due to the earth and of the normal force $\vec{F}_n$ due to the surface of the hill (since the frictional force $\vec{F}_f$ due to the surface is negligible). If the sled has a mass m, its equation of motion is then

$$m\vec{a} = \vec{F}_g + \vec{F}_n \quad \text{(E-1)}$$

The directions of these forces are indicated in Fig. E-1b where the gravitational force $\vec{F}_g = m\vec{g}$ is vertically downward and the normal force $\vec{F}_n$ is perpendicular to the surface of the hill.

► *Implementation*

Since the sled moves along the hill, we use the equation of motion Eq. (E-1) to relate component vectors parallel to the downhill direction $\hat{x}$. The component vector of the acceleration $\vec{a}$ parallel to $\hat{x}$ is simply equal to a since this acceleration is itself parallel to $\hat{x}$. The component vector of the gravitational force $\vec{F}_g$ parallel to $\hat{x}$ can be found from the vector diagram in Fig. E-1c and is $(F_g \cos\theta)\hat{x}$. The component vector

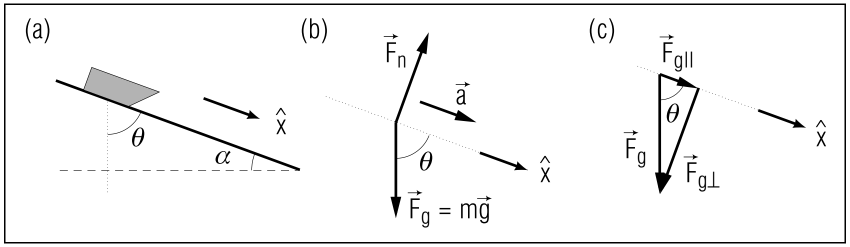


Fig. E-1: Sled sliding down a hill. (a) Sketch of the situation. (b) Diagram indicating the forces on the sled and its acceleration. (c) Component vectors of the gravitational force relative to $\hat{x}$.

of the normal force $\vec{F}_n$ parallel to $\hat{x}$ is zero since $\vec{F}_n$ is perpendicular to $\hat{x}$. Hence Eq. (E-1) implies for the component vectors parallel to $\hat{x}$ the equality

$$m\vec{a} = (F_g \cos\theta)\hat{x} + 0 = (mg\cos\theta)\hat{x}$$

so that

$$\vec{a} = g\cos\theta\hat{x} \tag{E-2}$$

► *Checking*

The result Eq. (E-2) makes sense since it implies that the acceleration of the sled is downhill and has a magnitude which is larger when the gravitational force is more nearly parallel to the surface of the hill (i.e., when θ is smaller). In the special case where the surface is horizontal so that $\theta = 90°$, gravitational force has no component parallel to the surface. Then $\vec{a} = 0$, as expected since a sled moving along a frictionless horizontal surface would move with constant velocity. Conversely, in the special case where $\theta = 0°$, the gravitational force is entirely parallel to the surface. Then $\vec{a} = \vec{g}$, as expected since the sled would then simply fall vertically downward under the sole influence of gravity.

► *Discussion*

According to Eq. (E-2), the acceleration $\vec{a}$ of the sled is independent of its mass m. This result is again a consequence of the fact that the gravitational force $\vec{F}_g$ is simply proportional to the mass m. Note that the component vector $\vec{F}_{g\parallel}$ of the gravitational force *parallel* to the surface of the hill makes the sled move downhill with constant acceleration. The component vector $\vec{F}_{g\perp}$ of the gravitational force *perpendicular* to

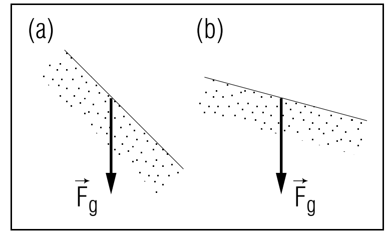


Fig. E-2.

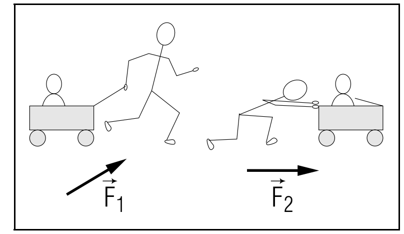


Fig. E-3.

the surface is balanced by the opposing normal force $\vec{F}_n$ due to the surface (so that $\vec{F}_{g\perp} + \vec{F}_n = 0$) to assure that the sled does not move perpendicularly to the surface of the hill. *

> * Fig. E-1a shows that $\theta + \alpha = 90°$ if α denotes the angle made by the slope of the hill with the horizontal. Thus $\theta = 90° - \alpha$ and the result Eq. (E-2) can also be written as $\vec{a} = g\sin\alpha\hat{x}$.

Illustration

E-1 Suppose the hill described in Example E-1 makes an angle $\alpha = 10°$ with the horizontal direction. What is the angle θ between the sled's path and the gravitational force? What is the sled's acceleration? What is the sled's velocity 2.0 sec after starting from rest? *(Answer: 113)*

Now: Go to tutorial section E.

Relating Motion to Component Forces (Cap. 3)

E-2 Consider again the sled described in Example E-1. Fig. E-2 shows two possible paths for the sled, one roughly along the gravitational force (Fig. E-2a) and one roughly perpendicular to the gravitational force (Fig. E-2b). For which path is the sled's acceleration larger? *(Answer: 121)*

E-3 A boy gives his friend a ride in a wagon which has well-oiled wheels and thus moves with negligible friction along the horizontal sidewalk (Fig. E-3). The boy first pulls the wagon, exerting on it a force $\vec{F}_1$, and then pushes the wagon, exerting on it a force $\vec{F}_2$. If $\vec{F}_1$ and $\vec{F}_2$ have equal magnitudes, for which force has the wagon a larger acceleration? *(Answer: 124)*

SECT.

F RELATING FORCES ON SEVERAL INTERACTING PARTICLES

Consider a system consisting of several interacting particles, e.g., particles joined together so that they are in contact. Then one can apply the equation of motion $m\vec{a} = \vec{F}$ successively to each of these particles. Furthermore, if the force $\vec{F}_{1,2}$ on some particle 1 due to some other particle 2 is not known, it can be found from the force $\vec{F}_{2,1}$ on particle 2 due to particle 1 since the mutual forces are reciprocally related so that $\vec{F}_{1,2} = -\vec{F}_{2,1}$. The following simple example illustrates how these principles can be systematically applied to such a system of interacting particles.

Example F-1: Force exerted on a rope supporting an object

During a mountain rescue operation, a person (of mass m_p) is suspended at rest from a long rope (of mass m_r) fastened to a hook anchored in a rock. What then is the force which must be exerted on the rope by the supporting hook?

Description: The situation is illustrated in Fig. F-1a. We know that gravitational forces act on both the person and the rope. We want to find the force $\vec{F}_{r,h}$ exerted on the rope by the hook.

Planning: We may first consider the rope and then the person. Since each of these objects remains suspended at rest, each has zero acceleration. Hence the equation of motion implies that the total force on each must be zero.

Consider first the rope. As indicated in Fig. F-1b, the forces acting on the rope are the downward gravitational force $m_r\vec{g}$ on the rope due to the earth, the force $\vec{F}_{r,h}$ on the rope due to the hook, and the force $\vec{F}_{r,p}$ on the rope due to the person. Hence the condition that the total force on the rope is zero implies that

$$m_r\vec{g} + \vec{F}_{r,h} + \vec{F}_{r,p} = 0 \tag{F-1}$$

Consider next the person. As indicated in Fig. F-1c, the forces acting on the person are the downward gravitational force $m_p\vec{g}$ on the person due to the earth and the force $\vec{F}_{p,r}$ on the person due to the rope. Hence the condition that the total force on the person is zero implies that

$$m_p\vec{g} + \vec{F}_{p,r} = 0 \tag{F-2}$$

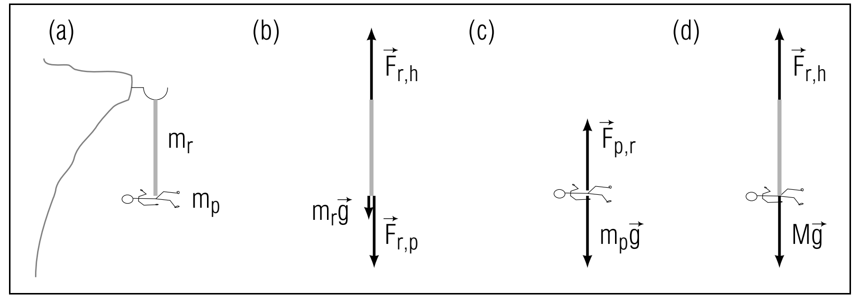


Fig. F-1: Person suspended from a rope fastened to a hook. (a) Sketch of the situation. (b) Diagram showing forces on the rope. (c) Diagram showing forces on the person. (d) Diagram showing forces on the combined object consisting of the rope and person.

In Eq. (F-1) the unknown force $\vec{F}_{r,p}$ on the rope due to the person is related to the force $\vec{F}_{p,r}$ on the person due to the rope by the reciprocal relation

$$\vec{F}_{r,p} = -\vec{F}_{p,r} \tag{F-3}$$

Implementation: To find the force $\vec{F}_{r,h}$ from Eq. (F-1), we need first to use our other relations to find the force $\vec{F}_{r,p}$. Thus Eq. (F-2) implies that $\vec{F}_{p,r} = -m_p\vec{g}$ so that Eq. (F-3) yields the result

$$\vec{F}_{r,p} = -\vec{F}_{p,r} = m_p\vec{g} \tag{F-4}$$

Hence Eq. (F-1) implies that

$$\vec{F}_{r,h} = -m_r\vec{g} - \vec{F}_{r,p} = -m_r\vec{g} - m_p\vec{g}$$

or

$$\vec{F}_{r,h} = -(m_r + m_p)\vec{g} \tag{F-5}$$

Checking: The result Eq. (F-5) makes sense since it claims that the force on the rope due to the hook is upward (i.e., opposite to $\vec{g}$) and has a magnitude equal to the sum of the weights of the rope and the person.

Discussion: We could also discuss the preceding problem by considering the rope plus the person as a single combined object having a mass

M. As indicated in Fig. F-1d, the forces on this combined object would then be the downward gravitational force $M\vec{g}$ due to the earth and the force $\vec{F}_{r,h}$ on the rope due to the hook. The condition that the total force on this combined object is zero implies then that

$$M\vec{g} + \vec{F}_{r,h} = 0 \text{ or } \vec{F}_{r,h} = -M\vec{g} \qquad \text{(F-6)}$$

By comparing this result with the previous result Eq. (F-5), we see that M must have the value $M = m_r + m_p$. In other words, the mass of the combined object consisting of the rope and the person must be equal to the sum of the masses of these objects. This is a very plausible result consistent with familiar observations.

TENSION FORCES PRODUCED BY LIGHT STRINGS

If the rope in the preceding example has a negligibly small mass it is called a "light" rope or string. Then $m_r = 0$ so that Eq. (F-1) implies that $\vec{F}_{r,h} + \vec{F}_{r,p} = 0$ or $\vec{F}_{r,h} = -\vec{F}_{r,p}$. Hence the tension forces exerted on the two ends of any light string (and thus also by the two ends of any such string) are always of equal magnitude. Thus a light string merely "transmits" the same force from one of its ends to the other; for example, the force exerted on the rope by the hook is simply the same as the force exerted on the person by the rope. The same statement is also true even if the light string is accelerated (since $m\vec{a} = \vec{F}$ for the string is still zero because its mass $m = 0$) and even if the string passes around a frictionless pulley (since the forces on opposite ends of any small portion of the string around the pulley must have the same magnitudes). The preceding comments can be summarized by this conclusion:

The magnitudes of the tension forces exerted by any two ends of a light string are equal.	(F-7)

Knowing About Tension Forces

F-1 The rope and pulley arrangement in Fig. F-2a is a simple "block and tackle," a device for lifting a heavy object suspended from the pulley. To see how it works, let us find the weight W of a box which can be supported (or lifted with constant speed) by a hand exerting an upward force of magnitude $\vec{F}_{r,h} = 100$ newton on the rope. The pulley is frictionless and both pulley and rope have negligible mass. (a) What are the magnitudes of the tension forces exerted by the rope on the hand and

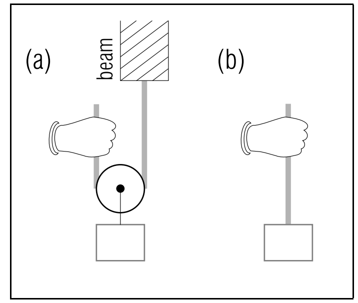


Fig. F-2.

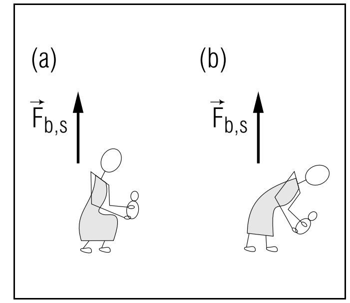


Fig. F-3.

on the supporting beam? What is the magnitude of the force on the rope due to the beam? (b) Consider as a particle the system comprising the box, rope, and pulley. What are the individual forces on this particle? Use the equation of motion to express W in terms of $\vec{F}_{r,h}$ and then to find a value for W. (c) For comparison, what is the weight W' of a box which can be directly supported at rest by a hand exerting an upward force of magnitude 100 newton (Fig. F-2b)? *(Answer: 114) ([s-4], [p-6])*

Applying the Equation of Motion to Several Objects (Cap. 4)

F-2 A small tractor at an airport pulls a train of two baggage carts so that all three objects have an acceleration of $1.5\,\text{m/s}^2$ toward the right. The loaded carts each have a mass of 300 kg and move with negligible friction. The tractor has a mass of 1,000 kg. (a) Apply the equation of motion to the last baggage cart (cart 2) in order to find the force on this cart due to the first cart (cart 1). (b) What is the value of the force on cart 1 due to cart 2? (c) Apply the equation of motion to cart 1 in order to find the force on this cart due to the tractor. *(Answer: 118) (Suggestion: [s-13])*

F-3 Young women often sustain back injuries while lifting children. To estimate the force causing such injury, consider a woman's upper body as a particle of mass 30 kg. The spine exerts on this body an upward force $\vec{F}_{b,s}$ (Fig. F-3a). As the woman lifts a child, suppose that both her upper body and the child move with an upward acceleration of magnitude $1\,\text{m/s}^2$. If the child has a mass of 20 kg (i.e., a weight of 44 pound), what is the force on the child due to the woman's upper body? What is the force $\vec{F}_{b,s}$? If this upward force due to the spine is directed roughly perpendicular to the spine (Fig. F-3b), the spine may be injured by excessive bending, or by strain of the muscles which prevent such bending. Injury is made less likely if the child is lifted as in Fig. F-3a, so that $\vec{F}_{b,s}$ is more nearly along the spine. *(Answer: 120) (Practice: [p-7])*

SECT.

G SUMMARY

DEFINITIONS

weight; Def. (B-2)

IMPORTANT RESULTS

Gravitational force on a particle due to the earth: Eq. (B-1)

$$\vec{F}_g = m\vec{g}$$

Contact forces: (Sec. C)

Depend on deformation of touching objects and tend to oppose this deformation.

(a) Force due to a spring: increases with magnitude of deformation and opposes this deformation.

(b) Tension force due to a string: along string *away* from particle on which it acts.

(c) Force due to a solid surface: $\vec{F}_\sigma = \vec{F}_n + \vec{F}_f$, where normal force $\vec{F}_n$ is away from surface and frictional force $\vec{F}_f$ is parallel to surface.

Tension forces produced by a light string: Rule (F-7)

The magnitudes of the tension forces exerted by any two ends are equal.

NEW CAPABILITIES

You should have acquired the ability to :

(1) Identify and describe the individual forces acting on a particle (Sects. A, B, and C, [p-1]).

(2) Systematically apply the equation of motion to find an object's mass, acceleration, or one of the forces acting on it when:

 (a) Individual forces are specified by arrow on a grid (Sec. C, [p-2], [p-3])

 (b) The acceleration and individual forces are all parallel or perpendicular to a given direction. (Sec. D and F, [p-4], [p-5])

(3) Relate qualitatively the motion of a particle along a straight path to the forces acting on it (Sec. E).

(4) Use the reciprocal relation between mutual forces to apply the equation of motion successively to particles moving together with the same acceleration. (Sec. F, [p-7])

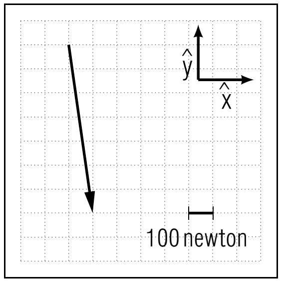


Fig. G-1.

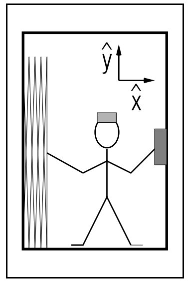


Fig. G-2.

Applying the Equation of Motion (Cap. 2)

G-1 In the process of walking, a 70 kg man presses (with his feet) downward and backward on the sidewalk surface. Thus he exerts on the sidewalk the force shown in Fig. G-1. Draw an arrow representing the force on the man due to the sidewalk. What is the man's acceleration? *(Answer: 116) (Suggestion: [s-3])*

G-2 While ringing a doorbell in a hall, a delivery man supports a package by pushing it horizontally against a wall so that the package remains at rest (Fig. G-2). He exerts on the package a horizontal force of magnitude equal to twice the weight of the package. If this weight is 50 newton, what are the frictional and normal forces exerted on the package by the vertical surface of the wall? *(Answer: 127) (Suggestion: [s-7])*

SECT.
H PROBLEMS

Relating Forces to Velocity

H-1 (This problem requires using the equation of motion with the relations between acceleration and velocity discussed in Unit 406.) A car's motion along a flat road surface depends on the force on the car due to the road surface. (This is also true for a man walking, as in problem G-1.) (a) Write an expression for the car's acceleration $\vec{a}$ in terms of its mass m and the frictional force $\vec{F}_f$ on the car due to the road surface. (b) If the car's tires are in good condition, and the road is dry, $\vec{F}_f$ has a maximum magnitude of about 0.60 times the car's weight. (This is the maximum frictional force possible without the tires slipping or skidding.) Use this value for $\vec{F}_f$ to find values for these quantities: (i) the maximum magnitude for the car's acceleration; (ii) the minimum time and the minimum distance required for the car to come to rest after traveling with an initial speed of 24 m/s (about 55 mile/hour); (iii) the maximum constant speed with which the car can travel around a flat (unbanked) curve of radius 150 meter. (c) If the road surface is icy, the maximum possible magnitude of $\vec{F}_f$ is only about 0.067 times the car's weight. Under these driving conditions, what are the values of the quantities listed in part (b)? *(Answer: 125) ([s-11], [p-8])*

H-2 *A traction device:* The device shown in Fig. H-1 exerts a force on the foot, thus keeping the ends of a broken leg bone aligned. To estimate this force, suppose that the device consists of a light string running over light frictionless pulleys. Consider as a particle the pulley and rope segments inside the rectangle shown in Fig. H-1. What is the magnitude of the tension force exerted by the string on the 5.0 kg metal "weight?" What is the sum of the forces exerted on the "particle" by the two string segments at A and B? What is the force exerted by this "particle" on the foot? *(Answer: 128) (Suggestion: [s-9])*

H-3 **Pushing a wheelchair up a ramp:* The orderly shown in Fig. H-2 exerts on a wheelchair the force $\vec{F}_O$ parallel to the ramp. The chair moves with negligible friction and with a constant velocity. (a) Express $\vec{F}_O$ in terms of known quantities, the mass m of the wheelchair and patient, and the angle θ between the ramp and the vertical direction.

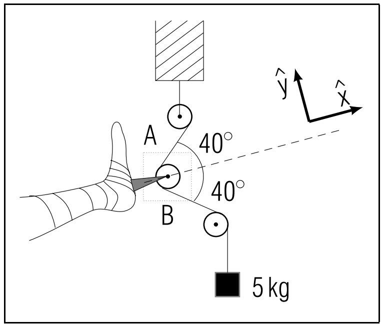


Fig. H-1.

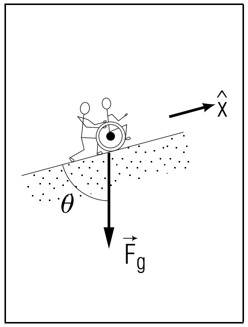


Fig. H-2.

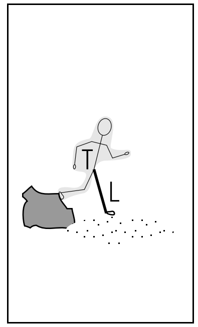


Fig. H-3.

(b) Does your result agree with these statements: Pushing a fat patient (for whom m is larger) requires a larger force than pushing a thin patient. Pushing a given patient up a steep ramp (for which θ is small) requires a larger force than pushing him up a less steep ramp. (c) What is the force $\vec{F}_O$ if the orderly pushes the chair with constant velocity *down* the ramp? *(Answer: 123) (Suggestion: [s-12])*

H-4 *Forces causing a broken ankle:* A hiker stepping off a large boulder lands on his left foot with the knee rigid (Fig. H-3). To estimate forces which commonly cause broken ankles, consider the following objects as particles. The particle T, of mass 55 kg, consists of the hiker's torso, head, arms and right leg. The particle L, of mass 10 kg, consists of the supporting left leg. As the left foot strikes the ground, the downward motion of these particles is stopped so that they both have approximately the same upward acceleration of magnitude $5.0 \times 10^2\,\mathrm{m/s^2}$. What is the force on the torso T due to its interaction with the left leg at the hip joint? What is the force on the left leg L due to its interaction with the foot at the ankle joint? (b) For comparison, what is the force on the left leg L due to the ankle joint if the hiker simply stands at rest on his left leg? (c) Compare these forces to explain why a broken ankle is a common hiker's injury, while hip injuries are more unusual. *(Answer: 126)*

Note: Tutorial section H contains further biological applications of the equation of motion.

TUTORIAL FOR D

SYSTEMATICALLY APPLYING THE EQUATION OF MOTION

d-1 *USING THE STRATEGY FOR PROBLEM SOLVING:* The capabilities of this unit include being able to *systematically* apply the equation of motion. (Cap. 2) This means applying the equation of motion according to some plan or strategy so that your problem solutions are clear and easy to check (both for you and for another person).

We recommend the strategy described in text section D. Try out this strategy as you work this problem:

An elevator, fully loaded with people, has a mass of 2.0×10^3 kg, and its maximum acceleration has a magnitude of $3.0\,\mathrm{m/s^2}$. The cable supporting the elevator can exert a force of magnitude 3.5×10^4 newton without breaking. What is the force exerted by the cable on the cab as the cab moves upward with its maximum acceleration? Is this force large enough to break the cable?

(1) Description

(a) Draw a diagram illustrating the situation.

(b) Summarize the known and desired information in terms of convenient symbols (including unit vectors to specify directions).

►

(2) Planning

(a) Decide on the principles to be used [$m\vec{a} = \vec{F}$].

(b) Choose the system to be considered [some specific particle].

►

(c) Express the principles in terms of known and desired information.

(1') Express $m\vec{a} = \vec{F}$ in terms of symbols for individual forces.

►

(2') Express the resulting equation in terms of symbols for known and desired quantities, using a diagram to indicate directions of all vector quantities.

►

(3) Implementation

(a) Solve the equations, expressing the desired quantities in terms of symbols for known quantities.

►

(b) Find the desired numerical values by replacing symbols by their known values (including units, signs, and directions).

►

(4) Checking

(a) Check that each step is correct.

(b) Check that the results are sensible.

Is the unit correctly a unit of force?

► ____________________

Is the magnitude reasonable when compared with the maximum force the cable can exert?

Is the direction correctly upward along the cable?

Now answer the final question in the problem.

Is this force large enough to break the cable?

► ______________________

(Answer: 18) Now: Return to text section D, and systematically solve problems D-1 through D-4.

TUTORIAL FOR E

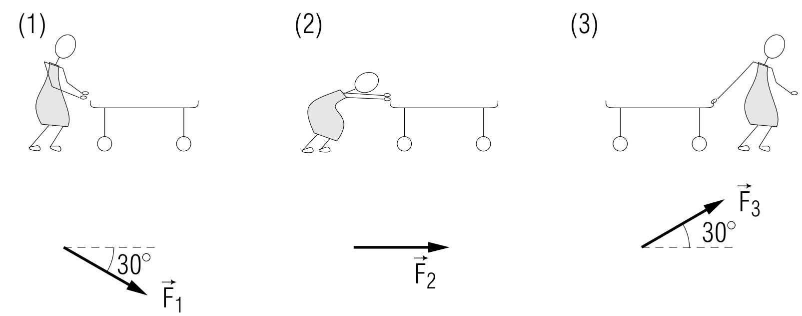
RELATING MOTION ALONG A STRAIGHT PATH

$\boxed{\textbf{e-1}}$ *PREDICTING MOTION ALONG A STRAIGHT PATH:* The discussion of the sled's motion in text section E illustrates a general means of predicting the motion of any particle which moves along a known straight path. Such motion depends only on component forces parallel to this path.

For example, consider a nurse pushing a wheeled stretcher which moves with negligible friction along a straight flat hallway as shown in the following drawing (1).

(a) List the objects which exert forces on this stretcher. Place a check beside each object which exerts a force with a nonzero component parallel to the stretcher's horizontal path.

►



The nurse can push or pull the stretcher in any of the ways shown in the preceding drawings. The corresponding forces which she can exert on the stretcher are indicated by arrows below the drawings.

(b) If all three possible forces have the same magnitude, for which of these forces is the component force along the path of *largest* magnitude? For which forces is this component force of *smallest* magnitude?

► Largest: $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$

► Smallest: $\vec{F}_1$, $\vec{F}_2$, $\vec{F}_3$

(c) Which of the methods shown for moving the stretcher will result in the largest acceleration of the stretcher?

► (1) (2) (3)

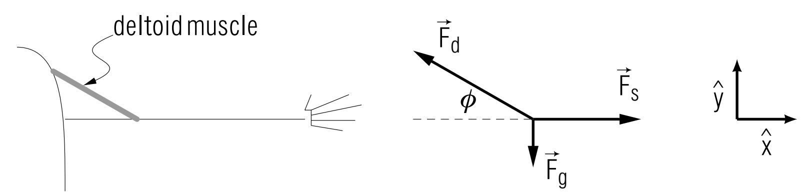
(Answer: 3) (Suggestion: Suggestion [s-14] reviews the process for finding component vectors.)) Now: Go to text problem E-2.

TUTORIAL FOR H

TWO BIOLOGICAL APPLICATIONS OF THE EQUATION OF MOTION

h-1 *FORCES CAUSING WEARING OF BODY JOINTS:* Recent work suggests that wear and degeneration of body joints (e.g., osteoarthritis) may be due to large forces exerted by these joints. One might suppose that the magnitude of the force exerted by a joint would be about equal to the weight of the body part it supports. (For example, the magnitude F_s of the force exerted by a shoulder joint would be about equal to the weight of the arm.) In fact, as the following problem illustrates, F_s can be much larger.

The following drawing shows a simple model of a shoulder joint with the arm horizontally outstretched. We approximate the force $\vec{F}_s$ exerted by the shoulder on the arm by assuming that it is directed along the arm, in this case horizontally. In addition, the arm is acted on by a force $\vec{F}_d$ due to the deltoid muscle, and by the downward gravitational force $\vec{F}_g$.



To find a value for F_s we shall apply the equation of motion to the arm, and then use the corresponding component equations to calculate F_s.

Write the equation of motion for the arm in terms of symbols for the individual forces on the arm. Then write the corresponding component equations parallel to $\hat{x}$ and $\hat{y}$.

► ______________________

____

____

Now: Check answer 16 and continue.

A typical man's arm has a mass of 3.9 kg and X-ray measurements of the deltoid muscles indicate that $\phi = 15°$.

What is the magnitude F_d of the force exerted by the deltoid muscle?

► $F_d =$ ____________________

What is the magnitude F_s of the force exerted by the shoulder joint on the extended arm?

► $F_s =$ ____________________

Compare F_s with the weight F_g of the arm by writing F_s as a number times F_g.

► $F_s =$ (______________)F_g

(Answer: 2)

h-2 *FORCES INVOLVED IN RUNNING:* When an animal runs with constant velocity, its body and head move with a nearly zero acceleration. Therefore the major effort in running is the exertion of muscle forces which bring each leg to rest as it strikes the ground, and then accelerate each leg as it leaves the ground so that it moves forward "catching up" with the body. The following *very approximate* application of the equation of motion suggests how an animal's speed is related to its size and shape.

At any instant of time, the total force on a leg is equal to the product ma of the leg's mass and acceleration. While the leg moves above the ground, this acceleration is approximately horizontal, and we can consider only the component equation of motion, $m\vec{a} = \vec{F}_m$, where $\vec{F}_m$ is the horizontal component force on the leg due to the animal's muscles.

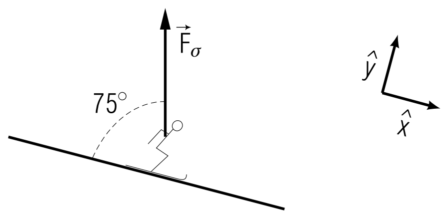
To approximate the acceleration $\vec{a}$, suppose that as the leg moves forward from rest, it has a constant acceleration until it reaches the speed V of the animal's body. Thus the change in the leg's velocity, $\Delta\vec{v} = \vec{V} - 0$, occurs during an approximate time of $(1/2)T$ where T is the time required for one stride. (During the second half of this stride, the leg moves forward, past the body, and slows down in preparation for striking the ground.) Thus $a = \Delta v/\Delta t = V/(T/2) = 2V/T$. The animal's speed V is also related to the time T of one stride by $V = d/T$, where d is the length of one stride.

Combine the relations $a = 2V/T, V = d/T$, and $F_m = ma$ in order to write an expression for an animal's speed V in terms of the magnitude

F_m of its component muscle force, the length d of its stride, and the mass m of its legs. (b) Animals such as deer, ostriches, racehorses, and greyhounds are all known for their large running speeds. Use your answer to part (a) to explain qualitatively how anatomical features common to all of these animals contribute to their running speeds. *(Answer: 13)*

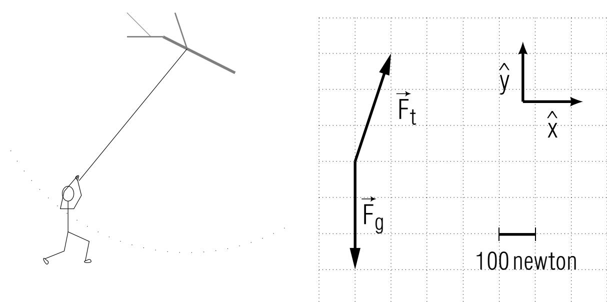
PRACTICE PROBLEMS

p-1 *FINDING COMPONENT SURFACE FORCES (CAP. 1):* The snow surface exerts on a skier a force $\vec{F}_\sigma$ which has a magnitude of 800 newton, and the direction shown in this drawing:



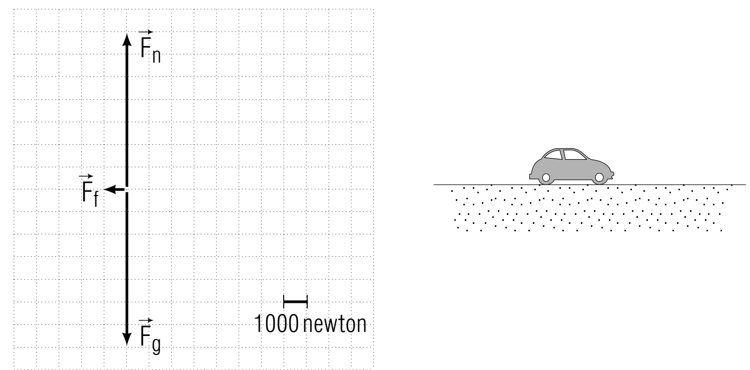
Find the frictional force $\vec{F}_f$ and the normal force $\vec{F}_n$ on the skier due to the snow surface. *(Answer: 1) Now: Return to text problem C-2 and make sure your work is correct.*

p-2 *APPLYING THE EQUATION OF MOTION (CAP. 2A):* While swinging on a rope, a child of mass 30 kg is acted on by the forces $\vec{F}_t$ due to the rope and $\vec{F}_g$ due to the earth shown in this diagram:



What is the child's acceleration at the instant shown? Check your answer by comparing the directions of the child's acceleration and path. Recall from Unit 406 that the acceleration of a particle moving along a curved path is directed towards the inside of the path. *(Answer: 11) (Suggestion: Review text problems C-5 and C-6.)*

p-3 *APPLYING THE EQUATION OF MOTION (CAP. 2A):* A car runs out of gas while traveling to the right on a level road. Until it comes to rest, the car is acted on by the gravitational force $\vec{F}_g$ due to the earth, and by the frictional and normal forces $\vec{F}_f$ and $\vec{F}_n$ due to the road surface. These forces have the values shown in this diagram:

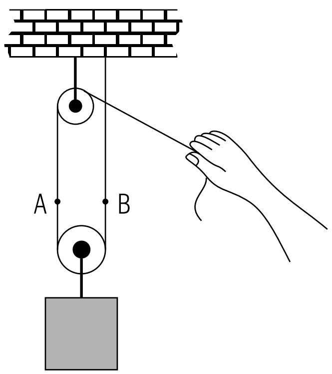


If the car's mass is 1.0×10^3 kilogram, what is its acceleration $\vec{a}$? If the car's initial velocity was 20 m/s towards the right, and its acceleration remains constant, what time interval is required for the car to come to rest? *(Answer: 8) (Suggestion: Review text problems C-5 and C-6.)*

p-4 *APPLYING THE EQUATION OF MOTION (CAP. 2B):* An elastic safety rope exerts on a falling climber a tension force of maximum magnitude 1.1×10^4 newton, independent of the height of the fall. (Such an elastic rope is less likely to injure a climber by jerking him with a large tension force at the end of a long fall.) If a climber has a mass of 100 kg, what is his acceleration $\vec{a}$ at the end of a vertical fall when he is acted on by the maximum upward tension force due to the rope? *(Answer: 10) (Suggestion: Review text problems D-1 through D-5.))*

p-5 *APPLYING THE EQUATION OF MOTION (CAP. 2B):* A man repairs a broken trailer hitch by fastening the trailer to his car with a horizontal length of chain with a "breaking strength" of 8.0×10^3 newton. (This breaking strength is magnitude of the maximum tension force the chain can exert without breaking.) The man plans to tow the trailer carefully, driving on flat horizontal roads with an acceleration of maximum magnitude $1.0\,\mathrm{m/s^2}$. If the frictional force on the trailer when in motion has a magnitude of 1.0×10^3 newton, what is the maximum mass of the loaded trailer which can be towed without breaking the chain? In other words, what is the mass of a loaded trailer which moves with an acceleration of magnitude $1.0\,\mathrm{m/s^2}$ when acted on by a tension force of magnitude 8.0×10^3 newton? *(Answer: 7) (Suggestion: Review text problems D-1 through D-5.)*

p-6 *KNOWING ABOUT TENSION FORCES:* Commonly a block and tackle consists of a light rope and light frictionless pulleys assembled as in the following drawing. Then pulling downward on the rope lifts the box suspended from the lower pulley.



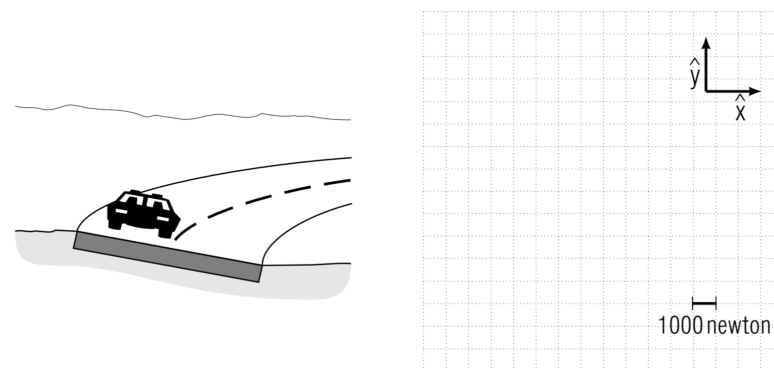
Suppose the hand shown exerts a force of magnitude 100 newton on the rope, thus supporting the box (or lifting it with a constant velocity). (a) What is the magnitude of the tension force exerted on the hand by the rope? (b) Consider as a particle the system comprising the box, lower pulley, and lower section of the rope between A and B. What is the tension force exerted on this particle by the rope extending upward from A? What is the tension force on this particle due to the rope extending upward from B? What is the weight W of the box? (c) If this block and tackle is used to lift a box having a weight of 500 newton, what is the magnitude of the force which the hand exerts on the rope? *(Answer: 12) (Suggestion: Review text problem F-1.)*

p-7 *APPLYING THE EQUATION OF MOTION TO SEVERAL OBJECTS (CAP. 4):* (a) While traveling along a flat level road, a driver applies his brakes so as to stop his car as rapidly as possible. His car, which has a mass m_c, is then acted on by a frictional force $\vec{F}_f$ due to the road surface. What is the car's acceleration $\vec{a}$?

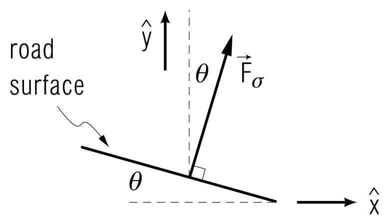
Now suppose that the car pulls a small trailer of mass m_T along the same road with the same speed. If the trailer has no brakes of its own, the frictional force on the trailer is negligible during the time the car's brakes are applied. However, both the car and trailer move with the same acceleration $\vec{a}\prime$, and the car is acted on by the same frictional force $\vec{F}_f$ described in part (a). (b) Write the equations of motion both for the car and for the trailer. Express the acceleration $\vec{a}\prime$ in terms of the frictional force $\vec{F}_f$ on the car and the masses of the car and trailer. (c) By comparing the accelerations $\vec{a}$ and $\vec{a}\prime$, briefly explain why it requires more time to stop the car with the trailer than to stop the car alone. *(Answer: 14) (Suggestion: Review text problems F-2 and F-3.)*

A More Difficult Practice Problem (Text Section H)

p-8 *RELATING FORCES TO VELOCITY:* The car shown in the following drawing travels away from the viewer along a road curving to the right. The car has a mass of 1.5×10^3 kg, and moves with a constant speed of 20 m/s along the circular road section which has a radius of 200 meter. (a) Express the force $\vec{F}_\sigma$ due to the road surface as a sum of two vectors with known values. Then use the following grid to construct an arrow representing $\vec{F}_\sigma$.



(b) To increase road safety, curves are often "banked," that is the road is inclined (as in the preceding drawing) so that its surface is approximately perpendicular to the force $\vec{F}_\sigma$.



Use your arrow representing $\vec{F}_\sigma$ to find the angle θ between the banked road surface and the horizontal direction. [Such a banked curve is safer than a flat curve, because the acceleration of a car is produced by a normal surface force rather than by a frictional force. When the road is wet, the frictional force may be very small (see text problem H-1), but the normal force does not depend on road conditions.] *(Answer: 17)*

SUGGESTIONS

s-1 *(Text problem D-4):* Write the equation of motion for the lamp in terms of the individual forces acting on it. Remember that the acceleration is zero for a particle moving with constant speed along a straight path.

s-2 *(Text problem C-6):* Remember that if a particle moves with constant speed v along a circular path of radius r, then the particle's acceleration $\vec{a}$ is v^2/r directed from the particle towards the circle's center.

s-3 *(Text problem G-1):* The force on the *man* due to the *sidewalk* has the same magnitude but the opposite direction as the force on the sidewalk due to the man. The total force on the man is the vector sum of the forces on him due to all objects with which he interacts. Here he interacts with the sidewalk and with the earth (through the gravitational force).

s-4 *(Text problem F-1):* Part (a): According to the reciprocal relation between mutual forces, the force on the *rope* due to the *hand* is equal in magnitude (and opposite in direction) to the force on the *hand* due to the *rope*. The ends of a light rope exert tension forces of the *same* magnitude.

Part (b): The particle described in Problem F-1 has an acceleration of zero. Thus its equation of motion is:

$$m\vec{0} = \vec{F}_{r,h} + \vec{F}_{r,b} + \vec{F}_g$$

where $\vec{F}_{r,b}$ is the upward force on the rope due to the beam, and $\vec{F}_g$ is the gravitational force of magnitude W.

s-5 *(Text problem D-1):* Application of the problem-solving strategy from text section D should include the following results.

Planning:

The equation of motion for the car is $m\vec{a} = \vec{F}_n + \vec{F}_f + \vec{F}_g$, where $\vec{F}_g = m\vec{g}$ is the downward gravitational force on the car.

Implementation:

The equation of motion implies these relations between component vectors parallel and perpendicular to the road:

$$m\vec{a} = \vec{F}_f,\ \vec{0} = \vec{F}_n + \vec{F}_g$$

s-6 *(Text problem A-2):* An object exerts a force on the child only if it is in direct contact with the child (i.e., if it touches the child), or if it exerts a long range force on the child (such as the gravitational force due to the earth). There is no force on an object (considered as a particle) due to itself, because when a particle is isolated, it moves with a constant velocity, and so the total force on the particle (due to itself) is $\vec{F} = m\vec{a} = \vec{0}$.

The tree branch does affect the child's motion. (If the branch weren't there, the child would fall.) However, the branch does not interact directly with the child, because it exerts neither a contact nor a long range force on the child.

s-7 *(Text problem G-2):* The equation of motion for the package is: $m\vec{a} = \vec{F}_g + \vec{F}_m + \vec{F}_f + \vec{F}_n$ where m and $\vec{a}$ are the package's mass and acceleration and the four forces on the package are the gravitational force, the force due to the man, and the frictional and normal forces due to the wall.

Draw labeled arrows indicating the directions of the five vector quantities appearing in the equation of motion.

►

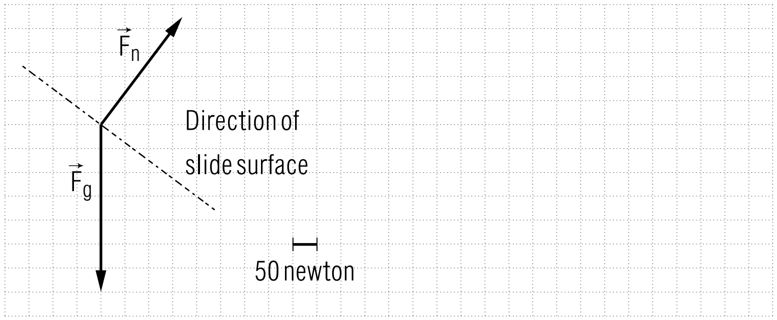
Because $m\vec{a} = \vec{0}$, the sum of the horizontal forces equals zero, and the sum of the vertical forces equals zero.

(Answer: 15) Now: Return to text problem G-2.

s-8 *(Text problem C-5):*

Use the following grid to construct an arrow representing the total force $\vec{F} = \vec{F}_n + \vec{F}_g$ on the child.

►



Express $\vec{F}$ as a magnitude with a direction relative to the slide surface. (Get the slide direction from the graph.)

► $\vec{F} =$ __________________

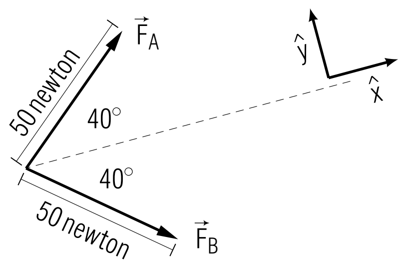
Use the equation of motion to find the child's acceleration $\vec{a}$.

► $\vec{a} =$ __________________

(Answer: 6) Now: Go to practice problem [p-2].

s-9 *(Text problem H-2):* All tension forces exerted by a single light string have the same magnitude. Thus here these forces have the magnitude 50 newton, the magnitude of the tension force of the string on the "weight" it supports.

Tension forces are directed from the particle on which they act, along the string. Thus the forces $\vec{F}_A$ and $\vec{F}_B$ exerted by the string segments at A and B have these values:



You can add these vectors most easily by first expressing each as a sum of component vectors parallel to $\hat{x}$ and $\hat{y}$.

Since the particle inside the dotted lines remains at rest, the total force on it due to the two string segments and due to the foot must be zero. Use this fact to find the force on the particle due to the foot. Then find the force on the foot due to the particle.

s-10 *(Text problem C-3):* Part (c): In describing forces on an object, we shall represent the object by a large dot, and draw from this dot arrows representing the forces. Use the following properties of forces to determine the directions of the three forces on the child: The normal force due to a surface is perpendicular to the surface and directed away from it. The frictional force due to a surface is parallel to the surface and has the direction opposite to the velocity of the parts of the object in contact with the surface. The gravitational force is directed downward.

s-11 *(Text problem H-1):* Part (b): The car's weight is the magnitude mg of the gravitational force. Therefore the frictional force has the magnitude $F_f = (0.60)\,\text{mg}$.

When you have found a value for the car's acceleration $\vec{a}$, apply the following relations from Unit 406 to find the remaining values:

Motion with constant acceleration along a straight path:

$$\Delta v = \vec{a}(\Delta t),\ \Delta r = \vec{v}_A(\Delta t) + 1/2\vec{a}(\Delta t)^2$$

Motion with constant speed along a circular path:

$$a = v^2/r$$

In applying the relations describing motion along a straight path, express vectors by using a unit vector parallel to the path. As a car's speed decreases, its acceleration has a direction opposite to its velocity.

s-12 *(Text problem H-3):* The equation of motion for the wheelchair is: $m\vec{a} = \vec{F}_O + \vec{F}_g + \vec{F}_n$, where $\vec{a} = \vec{0}$ is the chair's acceleration, and $\vec{F}_g$ and $\vec{F}_n$ are the gravitational and normal forces on the chair.

To find $\vec{F}_O$, which is parallel to the ramp, we proceed as in Example E-1 to use the equation of motion to relate component vectors parallel to the ramp (i.e., parallel to $\hat{x}$).

What are the component vectors parallel to $\hat{x}$ of the following forces:

► $\vec{F}_O$: ____________________

► $\vec{F}_g$: ____________________

► $\vec{F}_n$: ____________________

Write the wheelchair's component equation of motion relating quantities parallel to the ramp surface.

► ____________________

(Answer: 9) Now: Return to text problem H-3.

s-13 *(Text problem F-2):* Use the strategy discussed in text section D to successively apply the equation of motion first to cart 2 and then to cart 1. Check your work, making sure it includes the following information.

Description:

Convenient symbols for known and desired information:

	Cart 1	Cart 2
mass	m_1	m_2
acceleration	$\vec{a}_1$	$\vec{a}_2$
normal force due to ground surface	$\vec{F}_{1,n}$	$\vec{F}_{2,n}$
gravitational force	$\vec{F}_{1,g}$	$\vec{F}_{2,g}$
force on cart 1 due to cart 2	$\vec{F}_{1,2}$	- - -
force on cart 2 due to cart 1	- - -	$\vec{F}_{2,1}$
force on cart 1 due to tractor	$\vec{F}_{1,t}$	- - -

Planning:

Part (a): The equation of motion for cart 2 is:

$$m_2 a_2 = \vec{F}_{2,n} + \vec{F}_{2,g} + \vec{F}_{2,1}$$

Part (b): The equation of motion for cart 1 is:

$$m_1 a_1 = \vec{F}_{1,n} + \vec{F}_{1,g} + \vec{F}_{1,2} + \vec{F}_{1,t}$$

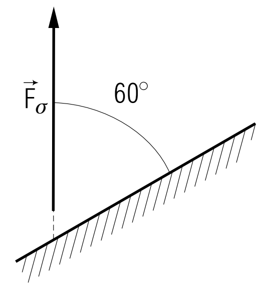
Although $\vec{F}_{1,2}$ is not initially known, it can be found from the value for $\vec{F}_{2,1}$ found in part (a).

s-14 *Text problem C-2, tutorial frame [e-1]):* The following discussion reviews the procedure (introduced in Unit 407) for constructing component vectors. To illustrate this procedure, let us find the component forces $\vec{F}_f$ and $\vec{F}_n$ parallel and perpendicular to a roof surface of the force $\vec{F}_\sigma$ due to that surface.

To find these component vectors, draw a line through the tail of $\vec{F}_\sigma$ and *parallel to the roof surface.* Draw a line through the tip of $\vec{F}_\sigma$ *perpendicular to the roof surface.* These lines should intersect in a right angle at point P. Then $\vec{F}_f$ can be drawn from the tail of $\vec{F}_\sigma$ to the point P, and $\vec{F}_n$ can be drawn from the point P to the tip of $\vec{F}_\sigma$.

Construct arrows representing the component vectors $\vec{F}_f$ and $\vec{F}_n$ parallel and perpendicular to the roof surface.

►



Use trigonometry to find the magnitudes of $\vec{F}_f$ and $\vec{F}_n$, and then express these vectors as multiples of the unit vectors $\hat{x}$ and $\hat{y}$.

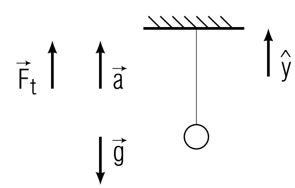
► $\vec{F}_f =$ ____________________

► $\vec{F}_n =$ ____________________

(Answer: 4) Now: Go to practice problem [p-1].

s-15 *Text problem D-3:* Application of the problem-solving strategy should include the following results. Vectors are specified by using the convenient upward unit vector $\hat{y}$.

Description:



$\vec{a} = 1.0\,\mathrm{m/s^2}\hat{y}$

$\vec{F}_t = 22\,\mathrm{newton}\hat{y}$

$\vec{F}_g = m\vec{g}$ (the gravitational force) desired: m

Planning:

The lamp's equation of motion is:

$$m\vec{a} = \vec{F}_t + \vec{F}_g$$

Expressing this equation in terms of symbols for known and desired quantities, we obtain:

$$m\vec{a} = \vec{F}_t + m\vec{g}$$

Implementation:

We wish first to find an algebraic expression for m.

$$m\vec{a} - m\vec{g} = \vec{F}_t$$

or

$$m(\vec{a} - \vec{g}) = \vec{F}_t$$

Hence magnitudes are related in this way:

$$m|\vec{a} - \vec{g}| = |\vec{F}_t|$$

or

$$m = \vec{F}_t / |\vec{a} - \vec{g}|$$

Use the directions of $\vec{a}$ and $\vec{g}$ to decide whether $|\vec{a} - \vec{g}|$ equals $(a + g)$ or $(a - g)$.

► $(a + g)$, $(a - g)$

(Answer: 5) Now: Return to text problem D-3.

ANSWERS TO PROBLEMS

1. $\vec{F}_f = -207\,\text{newton}\,\hat{x}$; $\vec{F}_n = 773\,\text{newton}\,\hat{y}$

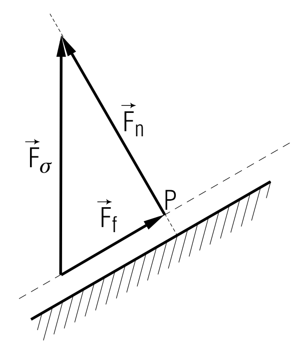
2. $F_d = 1.5 \times 10^2$ newton, $F_s = (1.4 \text{ or } 1.5) \times 10^2$ newton, $F_s = (3.6 \text{ or } 3.8) F_g$

3. a. forces due to gravity, hall floor, and nurse. Only force due to nurse has a non-zero component along the path.

   b. Largest: F_2; Smallest: F_1 and F_3

   c. (2)

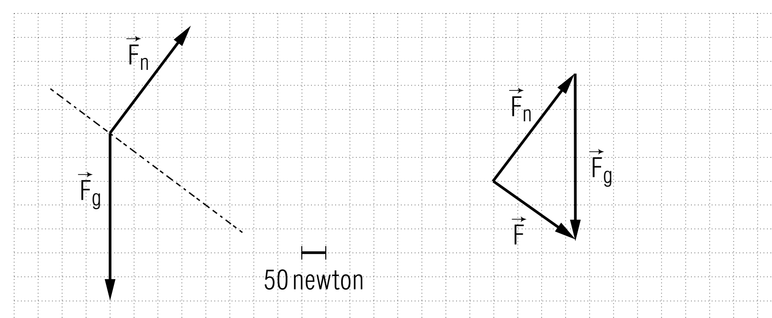
4.



$\vec{F}_f = 400\,\text{newton}\,\hat{x}$; $\vec{F}_n = 693\,\text{newton}\,\hat{y}$

5. $a + g$, because $\vec{a}$ and $\vec{g}$ are opposite in direction.

6.



$\vec{F} = 210\,\text{newton}$ downward along the slide, $\vec{a} = 6.0\,\text{m/s}^2$ downward along the slide.

7. $7.0 \times 10^3\,\text{kg}$

8. $\vec{a} = 1.0\,\text{m/s}^2$ towards the left, $20\,\text{sec}$

9. $\vec{F}_0$: $\vec{F}_0$; $\vec{F}_g$: $-\vec{F}(\cos\theta)\hat{x} = -mg(\cos\theta)\,\hat{x}$; $\vec{F}_n$: $\vec{0}$; $m\vec{a} = m\,\vec{0} = \vec{F}_0 + (-mg\cos\theta)\,\hat{x}$

10. $1.0 \times 10^2\,\text{m/s}^2$ upward

11. $\vec{a} = (3.3\,\text{m/s}^2)\,\hat{x}$

12. a. $100\,\text{newton}$

    b. $100\,\text{newton}$ upward, $100\,\text{newton}$ upward, $W = 200\,\text{newton}$

    c. $250\,\text{newton}$

13. a. $V = \sqrt{F_m d/(2m)}$

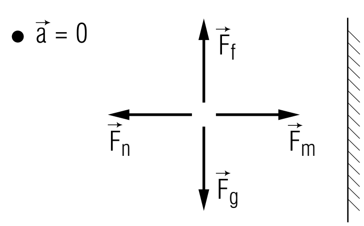
    b. All have long legs, resulting in a long stride length d, thin legs, resulting in a small mass m, and powerful muscles located in their body, where they do not increase the mass m of the legs.

14. a. $\vec{a} = \vec{F}_f/m$

    b. $m_C\vec{a}\prime = \vec{F}_{C,T} + \vec{F}_f$, $m_T\vec{a}\prime = \vec{F}_{T,C}$, where the mutual forces on the car and trailer due to each other are $\vec{F}_{C,T}$ and $\vec{F}_{T,C}$. [Each equation of motion may also include a gravitational force and a normal force due to the road, but for each particle the sum of these forces is zero.] $\vec{a}\prime = \vec{F}_f/(m_C + m_T)$

    c. Because $\vec{a}\prime$ is smaller than $\vec{a}$, the velocity of the car and trailer changes less rapidly than that of the car alone. Therefore the car and trailer require a longer time to come to rest.

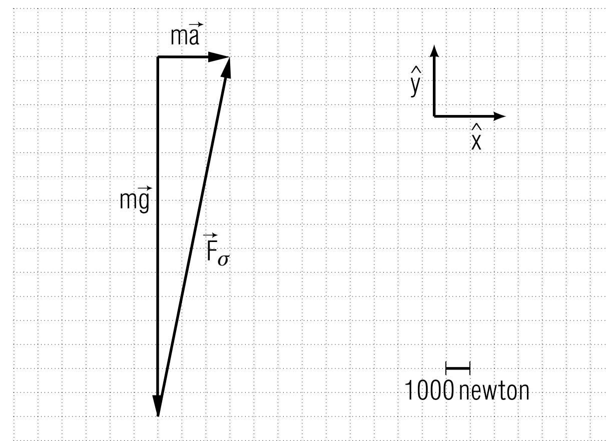
15.



The four forces are $\vec{F}_m$ due to the man, $\vec{F}_f$ and $\vec{F}_n$, the frictional and normal forces due to the wall, and $\vec{F}_g$ due to gravity. The frictional force must be parallel to the wall surface, and here it is upward.

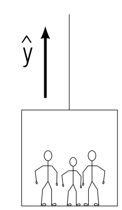
16. $m\vec{a} = \vec{0} = \vec{F}_s + \vec{F}_d + \vec{F}_g$; $\vec{0} = \vec{F}_s + (-F_d\cos\phi)\,\hat{x}$; $\vec{0} = \vec{F}_g + (F_d\sin\phi)\,\hat{y}$

17. a. $\vec{F}_\sigma = m\vec{a} - m\vec{g}$, where m is the mass of the car and $\vec{a}$ its acceleration. $\vec{a} = v^2/r$ toward the right, where v is the car's speed and r the radius of its path.



    b. $\tan\theta = 3/15$; $\theta = 11°$

18. *Description*:



Known: mass of elevator: $m = 2.0 \times 10^3\,\text{kg}$

acceleration of elevator: $\vec{a} = 3.0\,\text{m/s}^2\,\hat{y}$

maximum force exerted by cable: $\vec{F}_{max} = 3.5 \times 10^4\,\text{newton}\,\hat{y}$

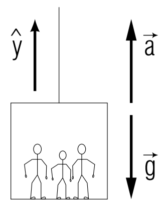
Desired: tension force due to cable: $\vec{F}_t$

Planning:

Consider the elevator as the particle.

$m\vec{a} = \vec{F}_g + \vec{F}_t$, where $\vec{F}_g = m\vec{g}$ is the downward gravitational force on the elevator.

$m\vec{a} = m\vec{g} + \vec{F}_t$. All quantities known except $\vec{F}_t$.



Implementation:

$\vec{F}_t = m\vec{a} - m\vec{g}$

$\vec{F}_t = (2.0 \times 10^3\,\text{kg})(3.0\,\text{m/s}^2\,\hat{y}) - (2.0 \times 10^3\,\text{kg})(-10\,\text{m/s}^2\,\hat{y})$

$\vec{F}_t = 2.6 \times 10^4\,\text{kg}\,\text{m/s}^2\,\hat{y} = 2.6 \times 10^4\,\text{newton}\,\hat{y}$

Checking:

yes, yes, yes

No, this force will not break the cable.

101. a. 10 newton downward
 b. 10 newton downward
 c. 5×10^5 newton downward or towards the earth's center

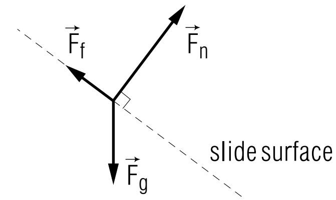
102. $\vec{F}_n = 693\,\text{newton}\,\hat{y}$; $\vec{F}_f = 400\,\text{newton}\,\hat{x}$

103. $4.0\,\text{m/s}^2$ toward the left, $20\,\text{m/s}$

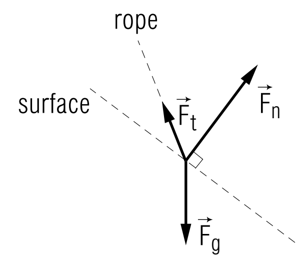
104. the earth, the surface of the hand, the surface of the notebook

105. a. earth, slide surface
 b. $\vec{F}_g$, $\vec{F}_f$, $\vec{F}_n$

 c.



106.



For the force due to the rope, you might choose $\vec{F}_t$ (tension force) or $\vec{F}_r$ (force due to the rope).

107. rope, earth

108. a. larger, to the left
 b. smaller, to the right

109. $6.0\,\text{m/s}^2$ downward along the slide surface

110. a. mass
 b. weight
 c. weight
 d. mass
 e. weight

111. $\vec{a} = \vec{F}_f/m$, $\vec{F}_n = -m\vec{g}$, or $m\vec{g}$ upward; $\vec{a} = 2.0\,\text{m/s}^2$ west; $\vec{F}_n = 1.5 \times 10^4$ newton upward

112. Weight: Stockholm; Mass: same for both.

113. $80°$, $1.7\,\text{m/s}^2\hat{x}$, $(3.4 \text{ or } 3.5)\,\text{m/s}\,\hat{x}$

114. a. 100 newton, 100 newton, 100 newton
 b. forces due to hand, beam, gravity; $W = 2F_{r,h} = 200$ newton
 c. 100 newton

115. frictional force: 1.8×10^2 newton $\hat{x}$; normal force: 1.5×10^3 newton $\hat{y}$.

116. The arrow should be equal in magnitude and opposite to the one shown in Fig. G-1: $-1.4\,\text{m/s}^2\,\hat{x}$

117. a. 53 newton $\hat{x}$
 b. -53 newton $\hat{x}$
 c. 60 newton $\hat{x}$

118. a. 450 newton toward the right
 b. 450 newton toward the left
 c. 900 newton toward the right

119. a. $m\vec{a} = \vec{F}_t + \vec{F}_g$, where $\vec{F}_g = m\vec{g}$ is the downward gravitational force on the lamp.
 b. $m = F_t/|\vec{a} - \vec{g}| = F_t/(a + g)$
 c. 2.0 kg

120. 220 newton upward, $\vec{F}_{b,s} = 550$ newton upward

121. Path (a)

122. 20 newton upward

123. a. $\vec{F}_0 = mg\cos\theta\,\hat{x}$
 b. yes, yes
 c. $\vec{F}_0 = mg\cos\theta\,\hat{x}$

124. $\vec{F}_2$

125. a. $\vec{a} = \vec{F}_f/m$
 b. 6.0 m/s^2, 4.0 sec, 48 meter, 30 m/s
 c. 0.67 m/s^2, 36 sec, 4.3×10^2 meter, 10 m/s

126. a. 2.8×10^4 newton upward, 3.3×10^4 newton upward
 b. 650 newton upward
 c. The force exerted at the ankle joint is larger than that exerted at the hip joint. (Also the ankle, being thinner, breaks more easily.)

127. frictional force: 50 newton $\hat{y}$; normal force: -100 newton $\hat{x}$

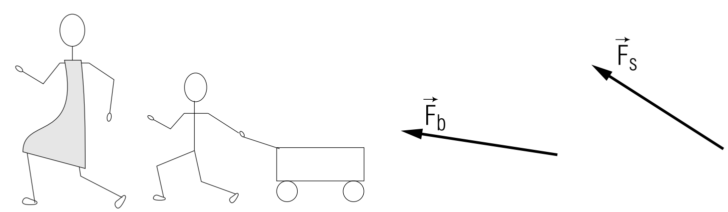
128. 50 newton, 76 newton $\hat{x}$, 76 newton $\hat{x}$

MODEL EXAM

1. **Measuring the mass of a birthday present.** An inquisitive boy wants to measure the mass of his birthday present (to see if it is an electric train or just some clothes). But because he has been forbidden to lift the box it is in, he decides to find its mass in the following way. He pulls the boxed present across the rug by using a spring scale which exerts a known horizontal force on the present. He finds that the frictional force exerted on the box due to the rug has a magnitude of 25 newton, and that a spring force of magnitude 28 newton is sufficient to give the box an acceleration of magnitude 0.5 m/s^2 along the rug.

 What is the mass of the boxed birthday present? *Show all your work.* Your solution should be sufficiently complete and systematic that it can be understood by another person.

2. **Forces involved in pulling a wagon.** A boy pulls a wagon by exerting on it the force $\vec{F}_b$ shown in the following diagram. The wagon moves with negligible friction along the horizontal sidewalk. His older sister then pulls the same wagon by exerting on it a force $\vec{F}_s$ which has the same magnitude as $\vec{F}_b$, but has the direction shown in the following diagram.



 Does the wagon have a larger acceleration along the sidewalk when the boy is pulling it or when the sister is pulling it?

Brief Answers:

1. Grader: look for evidence of using $m\vec{a} = \vec{F}_s + \vec{F}_f$, where $\vec{F}_s$ = force due to scale, $\vec{F}_f$ = frictional force. (May also include gravitational and normal forces.) Look for substitution of values such that $\vec{F}_s$ and $\vec{F}_f$ have opposite directions. Answer: 6 kg

2. the boy